

THE FERMAT CONJECTURE

by

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THE FERMAT CONJECTURE

FADE IN:

EXT. OXFORD GRAMMAR SCHOOL - DAY (1963)

A ten-year-old ANDREW WILES is running toward the school to talk with his friend, Peter.

INT. HALL OF SCHOOL

Andrew Wiles excitedly approaches PETER, who is standing in the hall.

ANDREW WILES

Peter, I want to show you something very exciting. Let's go into this classroom.

INT. CLASSROOM

Andrew Wiles writes on the blackboard:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11...positive integers.

3, 5, 7, 11, 13...prime numbers.

$$2^3 = 2 \cdot 2 \cdot 2 \quad 3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \quad x^n = \underbrace{x \cdot \dots \cdot x}_{n\text{-terms}}$$

and explains to Peter the Fermat Conjecture; as he writes on the blackboard.

ANDREW WILES

The Fermat Conjecture states that the equation  $x^n + y^n = z^n$  has no positive integer solutions, if  $n$  is an integer greater than 2.

Note that  $3^2 + 4^2 = 5^2$  so the conjecture is not true when  $n=2$ .

The famous French mathematician Pierre Fermat made this conjecture in 1637. See how beautiful and symmetric the equation is!

Peter, in a mocking voice, goes to the blackboard and writes on the blackboard:  $1963-1637=326$ .

PETER

Well, one thing is clear...the conjecture has been unproved for 326 years...you will never prove it!

ANDREW WILES

Don't be so sure. I have already shown that it is only necessary to prove it when  $n$  is a prime number and, besides, Fermat could not have known too much more about it than I do.

PETER

How did you learn about the conjecture?

ANDREW WILES

I found a book in the public library that was devoted entirely to it. Would you believe that even Einstein was not able to prove it!

PETER

And you actually think you are going to be able to prove it...?

ANDREW WILES

Well, I am certainly going to try...I find it absolutely astonishing that mankind has not been able to prove such a simple and elegantly stated problem.

PETER

I am afraid I do not share your enthusiasm. Why should anybody care whether or not the Fermat Conjecture is true or not? There are probably many such unsolved problems in mathematics.

ANDREW WILES

I guess it is just a matter of personal taste. For me the strong appeal of the problem is the simplicity and elegance of its statement contrasted with the apparent extreme difficulty of its solution.

PETER

How do you know it is even true?

ANDREW WILES

I don't, but according to the book, it has been proved for many special cases, and that is why I think it is reasonable to believe it is true.

PETER

Have you discussed the problem with anybody else?

ANDREW WILES

No, all that I know about the problem is what I read in that book. Since nobody can prove it, what could I possibly learn by talking to other people about it or reading more books about it?

PETER

Did the book indicate that Fermat had any ideas as how to prove his conjecture?

ANDREW WILES

He actually claimed that he had a beautiful proof, but the margin of the math book that he was reading was too small to contain it.

PETER

So, if I understand you correctly, to establish the Fermat Conjecture one just has to show that the equation  $x^p+y^p=z^p$  has no positive integer solutions when  $p$  is a prime number.

ANDREW WILES

When  $p$  is a prime number greater than 2.

INT. MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH -  
GERMANY - DAY (AUGUST 1984)

German mathematician GERHARD FREY is in the common room having tea with several mathematicians, one of which is MATHEMATICIAN #0.

MATHEMATICIAN #0

Gerhard, I understand you have a program for proving the Fermat Conjecture. Would you explain it to us?

Gerhard Frey goes to the blackboard in the common room and explains his program.

GERHARD FREY

Well, after very carefully studying the work of J.-P. Serre and Barry Mazur over a period of several years, I derived the following program for establishing the Fermat Conjecture. Suppose the Fermat Conjecture is false. Then there exists a prime number  $p$  and positive integers  $a, b, c$  such that  $a^p + b^p = c^p$ . Consider the elliptic curve  
 $E^*: w^2 = z^3 + (b^p - a^p)z^2 - a^p b^p z$ . Recall the Shimura-Taniyama Conjecture, which states that every elliptic curve is modular. Then, to prove the Fermat Conjecture: 1. Prove the Shimura-Taniyama Conjecture  
 2. Show  $E^*$  is not modular.

MATHEMATICIAN #0

In other words, if one can show your elliptic curve  $E^*$  is not modular, then to prove the Fermat Conjecture by your program, one would have to prove the Shimura-Taniyama Conjecture, which would immediately imply that  $E^*$  is modular.

GERHARD FREY

Yes, and since an elliptic curve cannot be both modular and not modular,  $E^*$  cannot exist so that the Fermat Conjecture must be true, for if it were false  $E^*$  must exist!

INT. CAFÉ STRADA, BERKELEY, CA - DAY (AUGUST 1986)

KEN RIBET and BARRY MAZUR are sipping cappuccinos while they discuss Ribet's work on Frey's program for the proof of the Fermat Conjecture.

KEN RIBET

I first heard about Frey's program when I was in Paris in January, 1985. J.-P. Serre gave me some ideas on how to show that Frey's elliptic curve  $E^*$  is not modular. I sent you a letter about my work to show that  $E^*$  is not modular, but you never replied.

Barry Mazur takes out a letter that had been sent to him by Ribet, and they review it together.

BARRY MAZUR

Well, I was a bit confused because you said you were only able to get a partial result, but actually you have established that Frey's elliptic curve  $E^*$  cannot be modular. Just add some extra gamma zero m structure and run through your argument, and it still works, and that gives you everything you need.

KEN RIBET

My God, you're absolutely right! But it will take me several months to get the bugs out and to very carefully write up my proof for publication.

BARRY MAZUR

What you have done is very remarkable, because before your work several well-known experts openly expressed doubts that Serre's approach to the problem would work. Also, your level-lowering principle for modular forms, that you establish to complete your proof, should have further important applications.

INT. KITCHEN IN PRINCETON HOME OF ANDREW WILES - DAY  
(AUGUST, 1986)

Andrew Wiles tells his wife NADA WILES that he is going to try to prove the Shimura-Taniyama Conjecture.

ANDREW WILES

I learned today at tea that Ken Ribet showed that the elliptic curve  $E^*$  that appears in the Frey program is not modular. And I was just electrified.

NADA WILES

Why is that so important?

ANDREW WILES

It is important because now the Fermat Conjecture can be attacked using very modern mathematical machinery. I knew at that moment the course of my life was changing... I am going to devote all of my effort to prove the Shimura-Taniyama Conjecture.

NADA WILES

How long will it take?

ANDREW WILES

It could take many months...even years, but even if I don't get it completely, there is a good possibility that I will get results that give some insight into that area of mathematics concerning elliptic curve theory.

NADA WILES

Andrew, you have tenure so even if you do not publish any more papers, your job will be secure so go for it, if that is what you really want to do.

ANDREW WILES

I knew I could count on your support.

INT. PRINCETON HOME ATTIC OFFICE OF ANDREW WILES - DAY  
(AUGUST 1986)

Andrew Wiles is working on the problem at his desk, which is piled high with books and papers. Although it is an attic office, it is actually the nicely finished third floor of his spacious house.

INT. FINE HALL OFFICE OF ANDREW WILES - DAY (AUGUST 1986)

Andrew Wiles is working on the problem at his desk, which is piled high with books and papers. He also is working on the blackboard in his office.

EXT. PATH AROUND LAKE CARNEGIE IN PRINCETON - DAY (AUGUST 1986)

Andrew Wiles walks along the path deep in contemplation.

INT. KITCHEN IN PRINCETON HOME OF ANDREW WILES - DAY  
(JANUARY, 1987)

Andrew Wiles gives a progress report to his wife, Nada.

ANDREW WILES

Last night I showed that I can use  
3-adic representations instead of  
2-adic representations.

NADA WILES

Why is that important?

ANDREW WILES

It gives me another degree of  
freedom...very important!

INT. YALE UNIVERSITY DEPARTMENT OF MATHEMATICS - DAY  
(JANUARY, 1988)

Andrew Wiles and MATHEMATICIAN A are in conversation in the hall of the Department.

MATHEMATICIAN A

Hello Andrew. What have you been  
up to these days?

ANDREW WILES

I have been concentrating my  
attention on the Shimura-Taniyama  
Conjecture.

MATHEMATICIAN A

But Andrew, that Conjecture  
immediately implies the Fermat  
Conjecture...nobody is going to  
prove that conjecture!

ANDREW WILES

Well, you asked me what I was  
doing, and I have told you!

MATHEMATICIAN A

Good luck, Andrew!

INT. KITCHEN IN PRINCETON HOME OF ANDREW WILES - DAY  
(JANUARY, 1989)

Andrew Wiles gives a progress report to his wife, Nada.

ANDREW WILES

I have been able to translate my  
previous ideas into ring-theoretic  
language.

NADA WILES

I do not have the slightest idea  
what you are talking about,  
Andrew.

ANDREW WILES

It means that I can now use all of  
the deep machinery associated with  
Hecke rings to attack the  
conjecture.

NADA WILES

For three years you have been  
working so hard that although I do  
not understand what you are doing,  
I do appreciate how very important  
it is because I understand you!

INT. FINE HALL OFFICE OF ANDREW WILES - DAY (FALL, 1989)

Andrew Wiles is meeting with his Ph.D. research student,  
R. RAMAKRISHNA, to give him a research problem to solve  
for his Ph.D. thesis.

ANDREW WILES

I am working on a research  
problem, which requires that I use  
the important results of Barry  
Mazur on deformations of Galois  
representations, but his results  
have to be extended to finite flat  
group schemes to be of use to me.  
If you can so extend his results,  
your work will satisfy your thesis  
requirement for the Ph.D. degree.

R. RAMAKRISHNA

I had a semester course on finite flat group schemes so I am confident that I can solve the problem you propose, but I have to read Mazur's papers, which are relevant to the problem.

ANDREW WILES

Great! Mazur's papers are very deep so do not hesitate to discuss them with me, if you need any help in understanding them.

R. RAMAKRISHNA

OK. What is the problem that you are working on that requires my results?

ANDREW WILES

Oh, I am just working on a classical problem concerning elliptic curves and Galois representations. As you know, I have published several papers in this area.

R. RAMAKRISHNA

Oh, I see.

INT. KITCHEN IN PRINCETON HOME OF ANDREW WILES - DAY  
(SPRING, 1991)

Andrew Wiles gives a progress report to his wife, Nada.

ANDREW WILES

A turning point in the entire proof has occurred. The relationship between the Hecke rings and the deformation rings can now be tested using two particular invariants.

NADA WILES

So this is a major breakthrough?

ANDREW WILES

Yes, I believe it is, even though many important details have to be worked out.

INT. FINE HALL - COMMON ROOM - PRINCETON UNIVERSITY - DAY  
(MAY, 1991)

MATHEMATICIAN B, MATHEMATICIAN C and MATHEMATICIAN D are discussing Andrew Wiles.

MATHEMATICIAN B

What has Andrew Wiles been doing mathematically for the past five years?

MATHEMATICIAN C

He has been teaching his courses, but I have not seen any research papers from him since 1985. Perhaps he has given up research.

MATHEMATICIAN D

Well, he is only thirty-eight years old. He is rather young for a man of his mathematical abilities to give up research.

MATHEMATICIAN C

Perhaps he has very high standards for publication, and he does not want to publish results that are not as important as some of his earlier results.

MATHEMATICIAN B

You may well be correct...he has solved several important problems in Iwasawa Theory.

INT. KITCHEN IN PRINCETON HOME OF ANDREW WILES - DAY  
(AUGUST, 1991)

Andrew Wiles gives a progress report to his wife, Nada.

ANDREW WILES

Unfortunately, I have to abandon the Hecke ring approach because I am just not able to get it to work.

NADA WILES

Does this mean you have to abandon the project after working so hard for some five years?

ANDREW WILES

No, not at all. I am now investigating some new, important results of Flach to be used to establish the relevant Euler system.

NADA WILES

It seems to me that after five years of very hard work you are now, in a sense, starting over.

ANDREW WILES

Actually, I am starting over on just the last part of the proof. I already have a very important result in the first part of the proof, which pertains to elliptic curves.

NADA WILES

So, even if you were to stop working on the conjecture now, your five years of work will not have been in vain?

ANDREW WILES

Exactly!

NADA WILES

Andrew, you have been working so hard for so long. Why don't you just stop here and publish your results. You will seriously damage your health, if you keep working so hard to the exclusion of everything else.

ANDREW WILES

As long as I can generate new ideas, I want to continue working on this fascinating problem. When the well dries up, I will stop, but not until then.

INT. FINE HALL OFFICE OF ANDREW WILES - DAY (JUNE, 1992)  
R.Ramakrishna informs Andrew Wiles that he has solved the problem that Wiles had proposed to him for his Ph.D. thesis.

K. RAMAKRISHNA

I believe I have solved the problem that you proposed to me in the fall of 1989.

ANDREW WILES

Excellent! Please write up your results very carefully, and I shall review them at the first opportunity.

K. RAMAKRISHNA

Should I give a copy to the other two members of my committee?

ANDREW WILES

It might be best if I review your results first; in case there are any serious errors in them.

K. RAMAKRISHNA

Fine...I should have a draft for you to read within three weeks.

INT. KITCHEN IN PRINCETON HOME OF ANDREW WILES (FALL, 1992)

Andrew Wiles gives a progress report to his wife, Nada.

ANDREW WILES

I believe that the end is near. Ramakrishna has completed his work successfully, and it appears that I shall be able to construct the relevant Euler system using Flach's techniques.

NADA WILES

That is truly great news! I was so afraid you might have hit a dead end when you dropped your previous approach to the final part of the proof.

INT. OFFICE OF ANDREW WILES FINE HALL PRINCETON UNIVERSITY - DAY (JANUARY, 1993)

Andrew Wiles takes friends and colleagues, NICK KATZ and PETER SARNAK, into his confidence. The scene opens with Wiles and Katz. Sarnak arrives late.

ANDREW WILES

Nick, I expect Peter Sarnak momentarily...Ah, here he is.

PETER SARNAK

I am sorry I am late.

ANDREW WILES

Peter, I have asked you and Nick Katz to join me today because I want to speak to you both in confidence. For the past seven years I have been working in total isolation on the Shimura-Taniyama Conjecture.

PETER SARNAK

Well, that explains why I haven't seen any papers published by you in recent years...thought you had given up research.

ANDREW WILES

I believe I have essentially completed the proof. I only have to finish the case when a certain Galois representation is reducible. However, I am uneasy about one portion of the proof, and I would like to check it with you, Nick.

NICK KATZ

I am up for it. What is the issue?

ANDREW WILES

In the course of the proof I have to construct a certain Euler system, and the construction is very delicate and very complicated.

NICK KATZ

I am available most afternoons this semester to check that part of the proof with you.

INT. FINE HALL OFFICE OF ANDREW WILES - DAY  
(JANUARY - MAY 1993)

Andrew Wiles and Nick Katz are working on the blackboard checking the proof.

INT. FINE HALL OFFICE OF NICK KATZ - DAY (JANUARY - MAY 1993)

Andrew Wiles and Nick Katz are working on the blackboard checking the proof.

INT. FINE HALL COMMON ROOM - DAY (JANUARY - MAY 1993)

Andrew Wiles and Nick Katz are discussing the proof.

INT. KATZ'S FINE HALL OFFICE - DAY (MAY 1993)

Nick Katz and Andrew Wiles have a final conversation on the checking of the proof.

NICK KATZ

Last night I again checked all of your calculations, and based on the several discussions we have had on the matter, I believe your construction of the Euler system is correct.

ANDREW WILES

Using some ideas in a paper by Barry Mazur I have finally been able to complete the case when a certain Galois representation is reducible. I will notify my former Ph.D. advisor, John Coates, that I would like to give a lecture on modular forms, elliptic curves and Galois representations at his conference in Cambridge England in June.

INT. THE HALL OF THE ISSAC NEWTON INSTITUTE, CAMBRIDGE UNIVERSITY, CAMBRIDGE, ENGLAND - DAY (JUNE 23, 1993)

LI GUO and RALPH GREENBERG discuss the forthcoming third and final lecture of Andrew Wiles.

LI GUO

Ralph, I missed the second lecture. Please fill me in.

RALPH GREENBERG

The second lecture was very technical and rumors are flying that Wiles is going to claim the Shimura-Taniyama Conjecture during his lecture today.

LI GUO

Let's go into the lecture hall so we can get a front row seat.

INT. LECTURE HALL - DAY

Andrew Wiles at the very end of his lecture announces that he has proved the Fermat Conjecture.

ANDREW WILES

...and so we have  $x^n + y^n = z^n$  has no positive integer solutions if  $n$  is greater than 2. I guess I should stop here.

AUDIENCE APPLAUSE. People take flash photographs.

EXT. FELLOWS GARDEN OF EMMANUEL COLLEGE - DAY

JOHN COATES, the Ph.D. advisor of Andrew Wiles, hosts a reception for Andrew Wiles and leads a toast to him. Champagne is served to approximately seventy people, who applaud during the toast.

JOHN COATES

To Andrew Wiles and his great accomplishment...

GUEST #1 walks up to Andrew Wiles and speaks to him.

GUEST #1

Now that the psychological barrier has been removed from the problem, there probably will be a simple proof of the Fermat Conjecture in a couple of years.

INT. IN THE HALL OF THE ISAAC NEWTON INSTITUTE - DAY  
(JUNE 24, 1993)

Ken Ribet is holding a press conference. REPORTER #1, REPORTER #2, REPORTER #3, REPORTER #4 and REPORTER #5 ask questions.

KEN RIBET

Andrew Wiles has left for Oxford to visit with his parents, and he asked me to answer your questions.

REPORTER #1

How long did he work on the problem?

KEN RIBET

He worked in total isolation on the problem for seven years... mostly in his attic office in his Princeton home.

REPORTER #2

How do we know that his proof is correct?

KEN RIBET

His paper will be divided up by Barry Mazur into six sections, and each section will be given to a separate referee for review.

REPORTER #3

When will a final decision be made as to whether or not the proof is correct?

KEN RIBET

Well, I would expect in three or four months, but it is a very complicated paper so it is not possible to say anything more precise.

REPORTER #4

Do you think he will receive the Nobel Prize for his proof?

KEN RIBET

There is no Nobel Prize for mathematics. Instead there is a Fields Medal, which is awarded every four years.

REPORTER #5

Do you think he will win the Field's Medal?

KEN RIBET

I don't know.

MONTAGE - THE WORLDWIDE REPORTING OF THE PROOF

--newspapers  
 --television  
 --radio  
 --magazines  
 --short wave radio

INT. SIMONYI HALL - INSTITUTE FOR ADVANCED STUDY (IAS) -  
 PRINCETON, NJ - DAY (JUNE 24, 1993)

MATHEMATICIAN #1 is working in his office when SECRETARY  
 #1 enters his office.

SECRETARY #1

You might be interested to know  
 that while I was driving to work  
 this morning I heard on the radio  
 that Andrew Wiles has solved a  
 very important problem.

MATHEMATICIAN #1

Really, do you know which problem?

SECRETARY #1

I do not recall, but the announcer  
 said that it was a very famous  
 problem.

Whereupon Mathematician #1 runs out of the building, gets  
 on his bicycle and rides about a mile to the kiosk on  
 Palmer Square and purchases a copy of the New York Times.  
 He then stands outside the kiosk in the hot sun reading  
 the article several times.

INT. COMMON ROOM - FULD HALL - INSTITUTE FOR ADVANCED  
 STUDY (IAS) - PRINCETON, NJ - DAY (JUNE 24, 1993)

Mathematician #1, MATHEMATICIAN #2, MATHEMATICIAN #3,  
 MATHEMATICIAN #4 are discussing at afternoon tea the  
 claim by Andrew Wiles of a proof of the Fermat  
 Conjecture. There is an atmosphere of great excitement.

MATHEMATICIAN #1

As soon as I heard the news I went  
 to the kiosk at Palmer Square and  
 purchased a New York Times. I was  
 captivated by the article, and I  
 just stood there reading the  
 article carefully several times  
 until there was simply nothing  
 more that I could wring out of it.

## MATHEMATICIAN #2

Since Wiles has such an impressive track record, his claim should be taken very seriously, indeed.

## MATHEMATICIAN #3

I received an email from Bombieri. He said that the proof that Wiles outlined at the conference appeared to him to be a solid attack on the problem.

## MATHEMATICIAN #4

In a sense this is a very depressing turn of events for most research mathematicians, for if Wiles' proof is ultimately found to be correct, no matter what he or she does, it will never measure up to a proof of the Fermat Conjecture.

INT. FINE HALL - COMMON ROOM - PRINCETON UNIVERSITY - DAY  
(JULY, 1993)

HAROLD SHAPIRO, the President of Princeton University, appears at afternoon tea and leads a champagne toast to Andrew Wiles. There are many students and mathematicians present.

## HAROLD SHAPIRO

To Andrew Wiles and his great achievement.

Several students and faculty members congratulate him.

EXT. PATH AROUND LAKE CARNEGIE - DAY (JULY, 1993)

Andrew Wiles is walking along the path, and he encounters a LAYPERSON (about 45-years old) walking in the opposite direction. As they pass, the layperson speaks to Andrew Wiles, who smiles in acknowledgement.

## LAYPERSON

Oh, I see you have solved a very important problem!

INT. FIELDS INSTITUTE FOR MATHEMATICAL RESEARCH -  
TORONTO, ONTARIO, CANADA - DAY (SEPTEMBER 1993)

KUMAR MURTY is addressing a conference of about fifty mathematicians. He is asked questions by MATHEMATICIAN #12 and MATHEMATICIAN #13.

KUMAR MURTY

Welcome to the 1993-94 academic year. As you know this year is supposed to be devoted to the study of  $L$ -functions. However, after Wiles made his claim in Cambridge in June we have decided to devote the fall term to the study of background material needed to understand his proof of the Fermat Conjecture.

MATHEMATICIAN #12

Will the talks and the lectures be published?

KUMAR MURTY

Yes, there will be a proceedings volume edited by me. I do not know, however, if it will contain all of the talks and invited lectures or just the invited lectures.

MATHEMATICIAN #13

What will be the subject of discussion during the spring term?

KUMAR MURTY

Hopefully, by then Wiles' manuscript will be available. If so, we will proceed to study it in detail.

MATHEMATICIAN #12

I understand from people who were at the Cambridge lectures, that a very large number of areas of mathematics are employed in his proof. Do you really think we will be able to understand the entire proof?

KUMAR MURTY

You raise a valid question. We will do the best that we can.

MATHEMATICIAN #13

This will be a very exciting year, indeed!

MATHEMATICIAN #12

Do you know of any other conferences being devoted to Wiles' work?

KUMAR MURTY

Professor Yau will be conducting such a conference at the Chinese University of Hong Kong from December 18<sup>th</sup> through December 21<sup>st</sup>.

MATHEMATICIAN #12

Will those talks and lectures be published?

KUMAR MURTY

Yes, Professor Yau and Professor Coates will be the editors of the proceedings volume. Professor Coates was the Ph.D. advisor of Andrew Wiles.

INT. PALACE OF FINE ARTS - SAN FRANCISCO - NIGHT (JULY 28, 1993)

A conference is being held to give the ubiquitous "intelligent layperson" some insight into what Andrew Wiles and others had done and the significance of their achievements. A panel discussion involving WILL HEARST (moderator), LENORE BLUM, JOHN CONWAY, LEE DEMBERT and Ken Ribet is in progress. The Palace is filled to capacity with more than 1,000 people in attendance.

WILL HEARST

A question that is frequently asked is just what is the value of the Fermat Conjecture? Why does anybody care that  $x^n + y^n = z^n$  has no positive integer solutions, if  $n$  is greater than 2?

LENORE BLUM

Well, a lot of modern algebra has been developed by very competent mathematicians in their attempt to prove the conjecture, and some of this algebra is used in engineering and computer science applications.

JOHN CONWAY

Actually, Wiles proved the Shimura-Taniyama Conjecture, and that is a very deep and important result in mathematics independent of the Fermat Conjecture, and it gives valuable insight into the Langlands Program that attempts to unite algebra and analysis.

LEE DEMBART

In fact, the Shimura-Taniyama conjecture states a very important property of elliptic curves, and those curves play a fundamental role in modern cryptology.

WILL HEARST

So attempts to solve what had become one of the greatest unsolved problems in all of mathematics with seemingly no practical applications whatsoever ultimately resulted in profound benefits to mathematics and to mankind.

KEN RIBET

The proof of the Fermat Conjecture as a milestone in the evolution of the intellect of mankind and, in particular, of the pursuit of pure thought, is of such magnitude and importance that I intend to write expository articles about it for the layperson.

INT. LECTURE HALL - GEORGE WASHINGTON UNIVERSITY - DAY  
(AUGUST 3, 1993)

Ken Ribet is giving a two-hour lecture on Wiles' proof of the Fermat Conjecture. He is standing in front of the blackboard in a large lecture hall filled to capacity.

KEN RIBET

My name is Ken Ribet. I am a Professor at Berkeley, and for the next two hours I will present an expository lecture on Wiles' proof of the Fermat Conjecture.

(MORE)

KEN RIBET (CONT'D)

I attended all three of his Cambridge lectures. My lecture will be essentially an elaboration of my expository article in the July/August issue of the Notices of the American Mathematical Society.

INT. LECTURE HALL - UNIVERSITY OF BRITISH COLUMBIA - DAY  
(AUGUST 15, 1993)

Barry Mazur is giving a forty-five minute lecture on Wiles' proof of the Fermat Conjecture. He is standing in front of the blackboard in a large lecture hall filled to capacity.

BARRY MUZUR

My name is Barry Mazur. I am a Professor at Harvard, and for the next forty-five minutes I will present an expository lecture on Wiles' proof of the Fermat Conjecture.

INT. OFFICE OF ANDREW WILES - FINE HALL - DAY (AUGUST, 1993)

Nick Katz comes into the office with a copy of Wiles' manuscript, a portion of which he is refereeing.

NICK KATZ

As you know I am refereeing the portion of your paper, which we reviewed together in great detail last spring.

ANDREW WILES

Yes.

Nick Katz places the manuscript on Wiles' desk, and he points to Theorem 15 in it.

NICK KATZ

Theorem 15 on page 137 plays a key role in your construction of the Euler system, and although last spring I thought I had followed your proof of the theorem, for some reason I am now not able to follow it, and in fact, I now believe it may not be true.

ANDREW WILES

OK, I shall review the proof and  
get back to you.

INT. PRINCETON HOME ATTIC OFFICE OF ANDREW WILES - DAY

Andrew Wiles is working on correcting the proof at his  
desk, which is piled high with books and papers.

INT. FINE HALL OFFICE OF ANDREW WILES - DAY

Andrew Wiles is working on correcting the proof at his  
desk, which is piled high with books and papers. He  
writes on the blackboard in his office.

INT. CBS STUDIOS, NYC - NIGHT (MARCH, 1986)

DAVID LETTERMAN is interviewing MARILYN VOS SAVANT, who  
has the highest IQ ever recorded.

DAVID LETTERMAN

Marilyn, tell us about yourself, a  
brief biography, if you will.

MARILYN VOS SAVANT

Well, I lived in St. Louis as a  
child, and I went to Washington  
University there for two years,  
where I studied philosophy.

DAVID LETTERMAN

What is all this business about  
your high IQ?

MARILYN VOS SAVANT

When I was ten years old my IQ was  
measured to be 228, the highest IQ  
ever measured.

DAVID LETTERMAN

I don't think you are so smart.

MARILYN VOS SAVANT

You are entitled to your opinion,  
David, but the fact remains;  
nobody has ever tested with a  
higher IQ.

DAVID LETTERMAN

So the people, who say that you are the smartest person in the world, are correct.

MARILYN VOS SAVANT

I have never made that claim. I value IQ tests as a measurement of a variety of mental abilities, but attempts to measure intelligence are useless; since intelligence itself involves so many factors.

INT. HARVARD OFFICE OF BARRY MAZUR - DAY  
(NOVEMBER, 1993)

Barry Mazur is discussing with Mathematician #12 the just published book by Marilyn vos Savant on the Fermat Conjecture.

MATHEMATICIAN #12

Barry, you seem to be upset.

BARRY MAZUR

Have you read the new book by Marilyn vos Savant, "The World's Most Famous Math Problem?"

MATHEMATICIAN #12

I saw it in the Harvard Coop, but when I observed that she claimed that Wiles' proof must be incorrect because of its use of non-Euclidean geometry, I decided not to purchase it.

BARRY MAZUR

Total nonsense! But what really annoys me is that in the book she thanks me for simply sending to her some material of mine that she requested by sending me a fax. It is very embarrassing.

EXT. PATH AROUND LAKE CARNEGIE IN PRINCETON - DAY

Andrew Wiles walks along the path deep in concentration and contemplation.

INT. OFFICE OF JOE KOHN - FINE HALL - DAY (DECEMBER 1, 1993)

JOE KOHN is the Chairman of the Princeton Mathematics Department. He discusses the status of the proof with Andrew Wiles.

JOE KOHN

Andrew, it has been over five months since you made your public claim of a proof of the Fermat Conjecture in Cambridge in June. What is the present status of your proof?

ANDREW WILES

Nick Katz has pointed out a problem in my construction of an Euler system, and I am trying to fix the problem.

JOE KOHN

It is all well and good that you work in total isolation, but there is a storm going on out there!

ANDREW WILES

There is no point in my making public statements until I have something of interest to say.

JOE KOHN

You are no doubt aware that ninety-nine percent of the time there is no recovery from an error in the attempted proof of a famous problem, so the odds are very strongly against you.

ANDREW WILES

Yes, I am aware of that fact.

JOE KOHN

I and many other faculty members are being inundated with requests for a copy of your manuscript and information on the status of your proof. Many people are claiming that you announced your claim of a proof of the Fermat Conjecture without properly and carefully checking your work first. You are causing a considerable amount of embarrassment to me and to the Department.

ANDREW WILES

What would you like me to do?

JOE KOHN

I want you to make a public statement to the mathematics community as to the status of your claim of a proof of the Fermat Conjecture.

ANDREW WILES

OK...I will do so.

INT. OFFICE OF KEN RIBET - UNIVERSITY OF CALIFORNIA -  
BERKELEY - DAY (DECEMBER 5, 1993)

Ken Ribet is discussing an e-mail that he just received from Andrew Wiles with MATHEMATICIAN #5.

MATHEMATICIAN #5

What is the status of Wiles' claim of the proof of the Fermat Conjecture?

KEN RIBET

Yesterday he sent an e-mail to me and to many mathematicians in the mathematics community in which he stated that a problem has arisen. Would you like me to read it to you?

MATHEMATICIAN #5

Yes, please do.

Ken Ribet reaches for his computer, locates the e-mail and proceeds to read it aloud.

KEN RIBET

The e-mail reads: "In view of the speculation on the status of my work on the Shimura-Taniyama Conjecture and the Fermat Conjecture I will give a brief account of the situation. During the review process a number of problems emerged, most of which have been resolved, but one in particular I have not settled.

(MORE)

KEN RIBET (CONT'D)

The key reduction of (most cases of) the Shimura-Taniyama Conjecture to the calculation of the Selmer Group is correct. However, the final calculation of a precise upper bound for the Selmer Group in the semi-stable case (of the symmetric square representation associated to a modular form) is not yet complete as it stands. I believe that I will be able to finish this in the near future using the ideas explained in my Cambridge lectures. The fact that a lot of work remains to be done on the manuscript makes it still unsuitable for release as a preprint. In my course in Princeton beginning in February I will give a full account of this work."

MATHEMATICIAN #5

It certainly does not sound very encouraging and, in a sense, rather embarrassing.

KEN RIBET

I had heard from people in other parts of mathematics that this whole algebraic geometry group has egg on its face, that we can't get our act together. They would say that we don't know a proof when we see one, so how could anyone believe what we say.

MATHEMATICIAN #5

Do you think he will fix the proof?

KEN RIBET

I am rooting for him, but as time goes by the unfilled gap in the proof is becoming something of an embarrassment. A proof that is unfinished is no proof at all.

MONTAGE - THE WORLDWIDE REPORTING OF THE  
ERROR IN THE PROOF

--newspapers  
--television  
--radio  
--magazines  
--shortwave

INT. FULD HALL - COMMON ROOM - DAY (DECEMBER 1993)

ANDRE WEIL and GERD FALTINGS, two very famous  
mathematicians, are discussing the gap in the proof  
during afternoon tea.

GERD FALTINGS

The very fact that Wiles is so  
competent means that he must be  
facing an extremely difficult and  
perhaps insurmountable problem.

ANDRE WEIL

I am willing to believe he has had  
some good ideas in trying to  
construct the proof, but the proof  
is not there. To some extent,  
proving the Fermat Conjecture is  
like climbing Everest. If a man  
wants to climb Everest and falls  
short of it by 100 yards, he has  
not climbed Everest.

GERD FALTINGS

Yes, strictly speaking Wiles'  
recent travails suggest that it  
wasn't a proof when it was  
announced.

EXT. PATH AROUND LAKE CARNEGIE - DAY (DECEMBER, 1993)

Andrew Wiles is walking along the path, and he encounters  
the same layperson that he previously encountered walking  
in the opposite direction. As they pass, the layperson  
speaks to Andrew Wiles, who smiles in acknowledgement.

LAYPERSON

Oh, I see you are having  
difficulties!

INT. DINING ROOM - IAS - DAY (JANUARY, 1994)

MATHEMATICIAN #6, MATHEMATICIAN #7, MATHEMATICIAN #8,  
MATHEMATICIAN #9, MATHEMATICIAN #10, MATHEMATICIAN #11  
are discussing rumors that they have heard.

MATHEMATICIAN #6

I heard that he had a nervous  
breakdown.

MATHEMATICIAN #7

Well, that would help to confirm  
my understanding that he has given  
up mathematics.

MATHEMATICIAN #8

According to a graduate student in  
Fine Hall, Wiles has corrected the  
proof.

MATHEMATICIAN #9

My understanding is that he has  
released his manuscript to certain  
people to work on fixing the  
proof.

MATHEMATICIAN #10

I heard that Gerd Faltings has  
corrected the proof.

MATHEMATICIAN #11

Indeed, that contradicts my  
understanding that Pierre Deligne  
has corrected the proof.

EXT. OUTSIDE FINE HALL - DAY (FEBRUARY 10, 1994)

Today Andrew Wiles is giving his first American lecture  
on his attempted proof of the Fermat Conjecture in Taplin  
Auditorium. There are several reporters and TV satellite  
trucks outside the building. The GUARD directs them all  
to leave.

GUARD

Ladies and gentlemen, please  
listen up. Andrew Wiles is  
conducting a class today in Taplin  
Auditorium and not a public  
lecture, so no reporters will be  
allowed in Taplin Auditorium, so  
please leave immediately.

All of the reporters and trucks start to leave as the  
scene ends.

INT. TAPLIN AUDITORIUM - FINE HALL - DAY (FEBRUARY 10, 1994)

Taplin Auditorium is filled to capacity with mathematicians from many institutions. The auditorium is charged with the excitement and anticipation of the moment. Andrew Wiles arrives late and sits in the front row. He is introduced by YUKOV SINAI.

YUKOV SINAI

It is my pleasure to introduce Andrew Wiles, who will speak on his work on the Fermat Conjecture.

Andrew Wiles walks slowly to the board and he equally slowly retrieves a piece of chalk. He turns to the audience and speaks.

ANDREW WILES

Well, in case any of you were wondering, I will say that I still haven't completely resolved the problem that arose in the review process.

That must be the understatement of the century. The AUDIENCE explodes with LAUGHTER. He then proceeds to explain in some technical detail the problem that has occurred in his proof.

ANDREW WILES

We know from the work of Frey, Serre and Ribet that the Frey curve  $E^*$  is not modular. So my task is to prove the Shimura-Taniyama Conjecture, which would establish that  $E^*$  is modular so that since  $E^*$  cannot be both modular and not modular,  $E^*$  cannot exist; so that the Fermat Conjecture must be true. I start with the Langlands-Tunnell Theorem, which gives me a certain modular lift. By an application of the deep result of Ribet, the so-called level lowering principle for modular forms, the rest of the proof reduces to showing that that lift leads ultimately to the proof of the Shimura-Taniyama Conjecture. In the course of my proof I construct two rings  $R$  and  
(MORE)

ANDREW WILES (CONT'D)

T, and using a theorem of Mazur I show that there exists a certain function  $f$  from  $R$  onto  $T$ . But I need to show that that function is one-to-one, and that is where the error in my proof occurs, because my construction of a certain Euler system is not correct.

When he finishes, several famous mathematicians surrounded Wiles at the blackboard. MATHEMATICIAN #12, who has been taking photographs throughout the lecture comments to MATHEMATICIAN #13.

MATHEMATICIAN #12

Look...Langlands, Bombieri, Deligne, Katz, Sarnak, Faltings, and Wiles...seven world-class mathematicians discussing a very serious approach to a proof of the Fermat Conjecture, and I am out of film.

MATHEMATICIAN #13

Such is the fate of the unthinking paparazzo!

INT. LECTURE ROOM 100 FINE HALL - DAY (FEBRUARY 1994)

Approximately thirty-five to forty research mathematicians are attending a lecture by Andrew Wiles on his attempted proof of the Fermat Conjecture. He is being relentlessly cross-examined by PIERRE DELIGNE, Nick Katz and Gerd Faltings. Andrew Wiles is standing at the blackboard.

ANDREW WILES

Now that the mathematical sightseers have dropped out, I have moved my Monday lectures this term from Taplin Auditorium to this room. Are there any questions related to any of my previous lectures?

PIERRE DELIGNE

Can you tell us precisely where the difficulty occurs in your proof?

ANDREW WILES

Well, the quick answer to your question is that my construction of a certain Euler system is defective. Since the construction is extremely technical, I have not yet been able to pinpoint exactly why the construction breaks down.

NICK KATZ

It might be helpful if you would explain again, as you did in your first lecture in Taplin Auditorium, the purpose of your Euler system construction.

ANDREW WILES

I first show that the Mazur deformation function  $f$  from the ring  $R$  into the ring  $T$  is an onto function. However, to complete the proof it is necessary to show that function is one-to-one. I use the Euler system that I am trying to construct to do this.

GERD FALTINGS

Have you tried any other way to show that the Mazur deformation function is one-to-one?

ANDREW WILES

Yes, initially I tried to do that using Iwasawa Theory and Hecke Ring Theory, but I was not able to satisfactorily manage the technicalities that arose.

PIERRE DELIGNE

Can you tell us precisely where the difficulty occurs in that approach?

ANDREW WILES

Unfortunately, I was not able to do that. I just got to a point where I could not proceed further so I decided to try the Euler system approach instead.

GERD FALTINGS

Do you think you might return to that approach?

ANDREW WILES

At this time I just want to concentrate on the Euler system approach, but I might again investigate my original approach, if I am not able to fix my Euler system construction.

NICK KATZ

If you ultimately do not get either approach to work, will you stop work on the problem and just publish the first part of your paper, which is correct and contains a very important result on elliptic curves?

ANDREW WILES

I don't know if I will try yet a third method to show the Mazur deformation function is one-on-one before releasing my manuscript.

INT. IN THE HALL OUTSIDE LECTURE ROOM 100 FINE HALL - DAY

Pierre Deligne and Gerd Faltings discuss the lecture.

PIERRE DELIGNE

What do you make of all of this? Do you think that Wiles is holding back or that he simply does not know why his two approaches do not work?

GERD FALTINGS

Andrew is a very honest person, so I tend to believe him.

PIERRE DELIGNE

Yes, but he is very competent, and it is hard to believe that he cannot be more explicit as to why his two approaches do not work.

GERD FALTINGS

Well, since he has not released his manuscript, I have no way to determine just how difficult these two approaches are. It is possible he fears the competition. I know I certainly would, if I were in his situation.

INT. PRINCETON HOME ATTIC OFFICE OF ANDREW WILES - DAY

Andrew Wiles is working on correcting the proof at his desk, which is piled high with books and papers.

INT. FINE HALL OFFICE OF ANDEW WILES - DAY

Andrew Wiles is working on correcting the proof at his desk, which is piled high with books and papers. He writes on the blackboard in his office.

EXT. PATH AROUND LAKE CARNEGIE IN PRINCETON - DAY

Andrew Wiles walks along the path deep in concentration and contemplation.

INT. PRINCETON HOME OF ANDREW WILES - DAY

Andrew Wiles and his wife Nada discuss his coping with the situation.

NADA WILES

Andrew, you have been very distant these past few weeks, and last night you were up most of the night pacing throughout the downstairs. Do you want to discuss the situation with me?

ANDREW WILES

Well, I am not at all concerned that there is a gap in the proof...sooner or later I will fix it. What concerns me is that somebody may steal my ideas and complete the proof before I do.

NADA WILES

Why do you think others are working on the problem...you have not released your manuscript.

ANDREW WILES

There are six copies of my manuscript in the hands of six separate referees, any one of whom could complete the proof. Further, sooner or later my manuscript will be leaked to others. It is simply not reasonable to think otherwise.

NADA WILES

Why don't you discuss the matter with Peter Sarnak? He will give you good advice.

ANDREW WILES

I am a very private person...I do not like to burden others with my concerns.

NADA WILES

Yes, but these are extraordinary circumstances.

INT. PRINCETON HOME OF PETER SARNAK - DAY (MARCH, 1994)

Andrew Wiles visits with Peter Sarnak to discuss the situation.

ANDREW WILES

Every morning I work for several hours on the problem, and think I have fixed it. But when I review my work in the evening, I find an error.

PETER SARNAK

This must be raising havoc with you, emotionally.

ANDREW WILES

Yes, I fear that I might mislead myself; I might spend years, or even decades going in circles trying frantically to fix the gap with methods that will never work.

PETER SARNAK

There are people who have spent thirty years on one problem... you can get caught in problems... trapped in them.

ANDREW WILES

It is getting harder and harder for me to think clearly. I am very tired. The pressure to fix the proof is enormous.

PETER SARNAK

You need somebody who is an expert in the subject, whom you can trust, to bounce ideas off of and to make suggestions to you...somebody to check every statement you make.

ANDREW WILES

But it has to be somebody who will let me call the shots. My greatest fear is that somebody will steal my ideas and finish the proof before I do. There are several very strong people out there, who could fix the proof...Katz, Deligne, Ribet, Faltings...just to name a few.

PETER SARNAK

What about your former Ph.D. student, Richard Taylor? He is an expert in the subject, and you can trust him.

ANDREW WILES

I have been very carefully considering him. He will be on leave next term from Cambridge University. He will be giving a colloquium lecture here next week. I will send him an e-mail tonight to ask him to consider the matter.

INT. COMMON ROOM - FINE HALL - DAY (MARCH, 1994)

Andrew Wiles and RICHARD TAYLOR are in discussion during afternoon tea.

ANDREW WILES

I enjoyed your colloquium lecture today. Did you review my e-mail to you concerning your working with me to fix the proof?

RICHARD TAYLOR

I would be honored to work with you. Would you be able to arrange for me to have an office and an apartment just for this term; as I have to return to Cambridge in the fall?

ANDREW WILES

There should be no problem  
obtaining accommodations for you.

RICHARD TAYLOR

I carefully reviewed your  
manuscript that you attached to  
your e-mail. Will we try to  
repair your construction of the  
Euler system or will we try a  
totally new approach to show that  
the deformation function is one-  
to-one?

ANDREW WILES

Originally I tried to show that  
the deformation function is one-  
to-one using Hecke rings, but I  
got stuck so I switched to the  
Euler system approach. Initially,  
we should try to get the latter  
method to work.

RICHARD TAYLOR

I shall review that method very  
carefully this evening.

ANDREW WILES

If we are not able to get that  
method to work, then I fear that  
we will have to develop an  
entirely new method to show that  
the deformation function is one-  
to-one, because I have exhausted  
all my ideas, over a period of two  
years, to get the Hecke ring  
approach to work.

INT. FINE HALL OFFICE OF ANDREW WILES - DAY

Andrew Wiles and Richard Taylor are working at the  
blackboard together trying to correct the proof.

INT. FINE HALL OFFICE OF RICHARD TAYLOR - DAY

Richard Taylor and Andrew Wiles are working at the  
blackboard together trying to correct the proof.

INT. FINE HALL COMMON ROOM - DAY

Andrew Wiles and Richard Taylor are discussing the problem with the proof.

EXT. PATH AROUND LAKE CARNEGIE IN PRINCETON - DAY

Andrew Wiles and Richard Taylor are walking along the path discussing the problem with the proof.

INT. OFFICE OF PETER SARNAK - FINE HALL - DAY (APRIL, 1994)

Peter Sarnak is being interviewed in his office by REPORTER #6 from the Princeton campus newspaper, The Daily Princetonian.

REPORTER #6

As you know, there is a lot of controversy associated with Professor Wiles, and the student paper, the Daily Princetonian, would like your views on the situation.

PETER SARNAK

Please be brief. I have to teach my class in a few minutes.

REPORTER #6

Do you think Professor Wiles is justified in not releasing his manuscript?

PETER SARNAK

Most certainly! That is so obvious that I do not wish to elaborate.

REPORTER #6

If he does not complete his proof, will all seven years of his work have been in vain?

PETER SARNAK

Absolutely not! Roughly speaking there are two parts to his paper, and there never was any problem with the proof of the first part.

(MORE)

PETER SARNAK (CONT'D)

In fact, if he had only announced that part in his Cambridge lectures and never even mentioned the second part, he would have received great recognition for that very important result.

REPORTER #6

What about the claim that he announced that he had proved the conjecture without first very carefully checking his proof?

PETER SARNAK

The problem is that Andrew is so private it is not generally known that he spent the entire 1993 spring term working with Professor Katz very carefully checking the most difficult part of the proof...the part that was ultimately found to be in error...I have to go to class now.

REPORTER #6

Thank you for your time, Professor Sarnak.

INT. WOLFENSOHN HALL - IAS - DAY (MAY 6, 1994)

Andrew Wiles is giving a general audience expository lecture on the theory of elliptic curves. He is introduced by PHILLIP GRIFFITHS, the Director of the IAS.

PHILLIP GRIFFITHS

Ladies and gentlemen, I present to you Andrew Wiles, who in the first part of his paper on the Fermat Conjecture has proved a very important and fundamental result on elliptic curves.

ANDREW WILES

I am supposed to be giving an elementary lecture on elliptic curves for the layperson, but since I have counted four Fields-Medal recipient laypersons in the audience, I will start my lecture by saying a few words about the gap in the proof.

(MORE)

## ANDREW WILES (CONT'D)

All known results and conjectures in the field point to the correctness of my claimed result, and I am confident that I will ultimately be successful in my attempt to resolve the thorny, relevant technicalities.

He then walks to the blackboard and starts writing on it as the scene ends.

## INT. FINE HALL OFFICE OF ANDREW WILES - DAY

Andrew Wiles and Richard Taylor are working at the blackboard together trying to correct the proof.

## INT. FINE HALL OFFICE OF RICHARD TAYLOR - DAY

Richard Taylor and Andrew Wiles are working at the blackboard together trying to correct the proof.

## INT. FINE HALL COMMON ROOM - DAY

Andrew Wiles and Richard Taylor are discussing the problem with the proof.

## EXT. PATH AROUND LAKE CARNEGIE IN PRINCETON - DAY

Andrew Wiles and Richard Taylor are walking along the path discussing the problem with the proof.

## INT. WILES' PRINCETON HOME ATTIC OFFICE - DAY (SEPTEMBER 19, 1994)

Andrew Wiles is sitting at his desk reviewing for the very last time why his construction of the Euler system fails, when he has a great and dramatic insight of how to get his original Hecke ring approach to work.

ANDREW WILES

Yes! Yes!

He immediately leaves his office and runs out of his house in the direction of the home of Peter Sarnak a block away arriving at the rear door of the house.

EXT. REAR DOOR OF PRINCETON HOME OF PETER SARNAK - DAY

PETER SARNAK

Andrew, come in.

INT. SARNAK'S HOME - DAY

ANDREW WILES

I just proved the Fermat Conjecture! I was sitting at my desk when suddenly, totally unexpectedly, I had this incredible revelation. It was the most, the most important moment of my working life. I decided to go back and look one more time at my construction of the Euler system to try and pinpoint exactly why it wasn't working, to try and formulate it precisely. I was trying, convincing myself that it didn't work, just seeing exactly what the problem was when suddenly, totally unexpectedly, I had this incredible revelation. I realized what was holding me up was exactly what would resolve the problem I had in my Hecke ring attempt three years earlier. It was so indescribably beautiful, it was so simple and so elegant, and I just stared in disbelief.

PETER SARNAK

Congratulations, Andrew. I am very pleased that you have finally resolved the difficulty. Well done!

INT. FINE HALL OFFICE OF ANDREW WILES - DAY (SEPTEMBER 19, 1994)

Andrew Wiles calls Richard Taylor, who by that time has returned to England, to tell him the good news.

ANDREW WILES

Richard, this is Andrew calling.

RICHARD TAYLOR(V.O)

I am glad you called. You would not believe the fantastic restaurant I discovered in Cambridge.

ANDREW WILES

Actually, I am calling about the problem.

RICHARD TAYLOR(V.O.)

The way they prepare salmon is superb. I have never tasted such salmon before!

ANDREW WILES

Richard, please, just listen...I have solved the problem!

RICHARD TAYLOR(V.O.)

Really...how did you do that?

ANDREW WILES

Well, I was reviewing our defective Euler construction, when I had this tremendous insight as how to get my original approach by Hecke rings to work.

RICHARD TAYLOR(V.O.)

Fantastic! Please e-mail to me your calculations so I can very carefully check them.

ANDREW WILES

I will write up the proof this evening and e-mail it to you.

RICHARD TAYLOR(V.O.)

Great! Congratulations! The end at last!

INT. TAPLIN AUDITORIUM - DAY (OCTOBER 31, 1994)

Andrew Wiles makes his first public statement on the correction to his proof. Since his intent to do this here and now was so poorly announced, there are only approximately thirty-five people present, none of which are reporters. Andrew Wiles is on stage at the podium.

ANDREW WILES

Gerd Faltings of the Max Plank Institute in Germany has very carefully refereed my corrected proof, and not only did he find it to be correct, but also he was able to somewhat simplify my corrections to the proof. I plan to release the manuscript in the near future. Thank you.

APPLAUSE as Andrew Wiles walks off the stage and out of the auditorium.

INT. PRINCETON HOME OF ROBERT LANGLANDS - DAY (DECEMBER 3, 1994)

A party is being conducted in honor of mathematician, Jim Arthur, who was the first Ph.D. student of ROBERT LANGLANDS, a very famous Professor at the Institute for Advanced Study. Approximately thirty-five people are present. MATHEMATICIAN #14 and Robert Langlands are in conversation in a corner of the living room.

MATHEMATICIAN #14

These are exciting times in mathematics. Where is Andrew Wiles?

ROBERT LANGLANDS

It is my understanding that he is presently on a train to Boston; as he will be giving a lecture at Harvard in two days.

MATHEMATICIAN #14

The Langlands-Tunnell Theorem plays a fundamental role in his proof. You must be very proud of your work.

ROBERT LANGLANDS

Yes, of course, but the work of many mathematicians went into Wiles' proof.

MATHEMATICIAN #14

I understand that the 1995-96 academic year at the Institute will be devoted to analyzing Wiles' proof.

ROBERT LANGLANDS

Yes, in fact, this had been decided before Wiles announced that the gap in his proof had been closed.

MATHEMATICIAN #14

Interesting. Tell me, how did you and Tunnell arrive at your theorem?

ROBERT LANGLANDS

When I wrote my monograph on base change, I included some examples. When Jerry Tunnell wrote his paper, he was providing yet another important example, employing the results in my monograph, the results of several other mathematicians together with some of his own original ideas.

INT. SCIENCE CENTER B - HARVARD UNIVERSITY - DAY  
(DECEMBER 5, 1994)

The very large auditorium is filled to capacity. This is Andrew Wiles' first lecture on the corrected proof outside of Princeton. He is introduced by Barry Mazur.

BARRY MAZUR

Ladies and gentlemen, I welcome you to Harvard's elementary number theory seminar...I give you Andrew Wiles.

Andrew Wiles walks to the blackboard, turns to the audience and is greeted with a very prolonged, standing ovation. He has a broad smile on his face as the scene fades.

INT. LECTURE HALL - HUNTER COLLEGE - DAY (DECEMBER 10, 1994)

Andrew Wiles delivers the Dolciani Lecture. There are approximately two hundred people present. At the beginning of his lecture there is a very animated question and answer session. MATHEMATICIAN #15, MATHEMATICIAN #16 and MATHEMATICIAN #17 ask him questions. He is introduced by JOSEPH ROITBERG, who is standing at the podium.

JOSEPH ROITBERG

Professor Dolciani was a long time member of the Hunter College faculty. She is best known as a mathematics educator. Professor Wiles of Princeton University has kindly agreed to deliver this year's Dolciani Lecture.

Roitberg leaves the podium and Andrew Wiles walks to the podium.

ANDREW WILES

I am honored to present the 1994 Dolciani Lecture. Professor Roitberg invited me to speak before I had corrected the proof. Before I begin my lecture, I am willing to entertain some questions that you might have.

MATHEMATICIAN #15

Is it true that you seriously tried to prove the Fermat Conjecture when you were ten years old?

ANDREW WILES

It is true that I had a fascination with the Fermat Conjecture when I was ten years old. However, I did not seriously pursue it until Ribet linked Fermat to Shimura-Taniyama.

MATHEMATICIAN #16

What important unsolved problem should mathematicians now attempt?

ANDREW WILES

The Riemann Hypothesis. I think most mathematicians would agree with me.

MATHEMATICIAN #17

Do you think Fermat had a proof of his conjecture?

ANDREW WILES

Fermat was in the habit of challenging English mathematicians with problems that he had solved. The fact that he never challenged them with his conjecture indicates that he probably did not have a proof. We understand Fermat's methods so well now that we can safely assume that he didn't find some completely new approach to it. If he had a proof, then it must have been a very, very cunning use of methods we know well, and the idea that only he could see it with methods known and used for two-hundred years is pretty hard to accept. Nevertheless, we still have to allow the possibility that he did, indeed, have a proof.

MATHEMATICIAN #16

Your work is considered to be the first significant advance in the Langlands Unification Program. Do you intend to work further on that program?

ANDREW WILES

At this time I do not, but that is not to say that working on that program is not worthwhile. It is extremely difficult to get significant results in that program.

MATHEMATICIAN #15

Why did you work in total isolation on the problem for seven years?

ANDREW WILES

Certainly for the first several years I had no fear of competition.

(MORE)

ANDREW WILES (CONT'D)

I simply didn't think I or anyone else had any real idea how to do it, but I realized after a while that talking to people casually about Fermat was impossible because it just generates too much interest, and you can't really focus yourself for years; unless you have this kind of undivided concentration, which too many spectators will have destroyed.

MATHEMATICIAN #17

Were you embarrassed by the discovery of the flaw?

ANDREW WILES

Why should I feel embarrassed? I'd made a sensational breakthrough. I'd opened the door on the subject. Even if I'd never fixed it I'd made an enormous advance. I'd removed the mystery from this problem. Also, the first part of my paper was correct, and in that part I established a very important new result on elliptic curves.

MATHEMATICIAN #15

How would you describe your experience of doing mathematics?

ANDREW WILES

Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. One goes into the first room and it's dark, completely dark. One stumbles around bumping into the furniture and then gradually you learn where each piece of furniture is, and finally after six months or so you find the light switch. You turn it on; suddenly it's all illuminated, you can see exactly where you are.

INT. OFFICE OF ALIX WILES - CAMBRIDGE, ENGLAND - DAY  
(MARCH, 1995)

ALIX WILES, the psychologist sister of Andrew Wiles, is being interviewed by IAN KATZ, a science writer for the British newspaper, The Guardian. Alix Wiles is seated at her desk. Ian Katz is sitting in a chair to her left.

IAN KATZ

As I explained on the phone, I am Ian Katz, a science writer for The Guardian, and I am writing a feature article on your brother's great achievement.

ALIX WILES

Yes.

IAN KATZ

You are a psychologist by training?

ALIX WILES

That is correct.

IAN KATZ

Do you have any insight as to how your brother was able to solve such a difficult problem?

ALIX WILES

I believe his breadth of interest was a major factor to his success.

IAN KATZ

Would you elaborate?

ALIX WILES

Well, in college he insisted on doing English A-level to break up the standard scientist's quartet of double mathematics, physics and chemistry. Also, he played cricket and tennis for his school. This breadth of interest enabled him to pull together fields and open up things that other people couldn't see.

IAN KATZ

I had heard that when the error was found in his proof, he greatly feared that others would complete his proof.

## ALIX WILES

Andrew's biggest fear was Gerd Faltings. He believed from the beginning that he was hard at it. He also thought that Ribet and Mazur might be hard at work trying to find a way to fix the proof.

INT. ROOM 220 DURHAM ENGINEERING BUILDING - YALE UNIVERSITY - DAY (APRIL 24, 1995)

Andrew Wiles is standing at the blackboard. The room is filled to capacity with approximately two-hundred people present.

## ANDREW WILES

When I first started to work on Fermat, I casually mentioned that fact to a member of Yale's Mathematics Department, and based on that experience I came to the conclusion that it would be best that I not continue to tell people that I was working on the problem.

INT. ROOM 1302 - COURANT INSTITUTE OF MATHEMATICAL SCIENCES - DAY (APRIL 24, 1995)

Andrew Wiles is standing at the blackboard. The room is filled to capacity with approximately two-hundred people actually in the room and an additional twenty-five or so people standing in the hall.

## ANDREW WILES

I count six or seven of the world's foremost authorities on differential equations sitting together in the front row, so I feel a bit like a fish out of water talking today about algebraic geometry and homological algebra.

INT. SCHAPIRO HALL AUDITORIUM - COLUMBIA UNIVERSITY - DAY (APRIL 27, 1995)

The very large auditorium is filled to capacity. He is introduced by DORIAN GOLDFELD.

DORIAN GOLDFELD

Ladies and gentlemen, welcome to  
Columbia University...I give you  
Andrew Wiles.

Andrew Wiles walks to the blackboard, turns to the audience and is greeted with a very prolonged, standing ovation. He has a broad smile on his face as the scene fades.

INT. HAROLD HELM AUDITORIUM - MCCOSH HALL - PRINCETON  
UNIVERSITY - NIGHT (MAY 15, 1995)

Andrew Wiles is delivering a general audience lecture to a large group of undergraduates at Princeton University. He is introduced by Joe Kohn, who is standing at the podium.

JOE KOHN

It is my pleasure to introduce to  
you Professor Andrew Wiles, who  
will talk to you about the Fermat  
Conjecture.

Joe Kohn leaves the stage as Andrew walks to the podium.

ANDREW WILES

Cubum autem in duos cubos, aut  
quadrato - quadratum in duos  
quadrato - quadratos, et  
generaliter nullam in infinitum,  
ultra quadratum potestatem in duos  
ejusdem nominis fas est dividere;  
cujus rei demonstrationem  
mirabilem sane detexi. Hanc  
marginis exiguitas non caperet.

...on the other hand...

INT. MAIN LECTURE HALL - HILL CENTER - RUTGERS UNIVERSITY  
- DAY (FALL, 1995)

GERALD TUNNELL, a Professor of Mathematics at Rutgers University is teaching a semester course on the proof of the Fermat Conjecture. This is the first day of the class. After he verbally gives an overview of the course, he more specifically outlines the course on the blackboard. He is standing at the black board with chalk in his right hand and the course outline in his left hand.

## GERALD TUNNELL

This course will cover topics in number theory, which lead to a proof of the Fermat Conjecture. Many of the most technical portions of the proof will be treated in a concurrent seminar. Topics will include some of the following:

1. Relation of integer solutions of  $x^n+y^n=z^n$  to the existence of elliptic curves with special properties.
2. Congruence for modular forms and Ribet's descent argument.
3. Modular elliptic curves.
4. Deformations of modular forms and Galois representations.
5. Ramification properties of Galois representations.
6. Group cohomology and Selmer groups.
7. Geometry and number theory of modular curves.
8. Congruence properties of modular forms.
9. Open problems in elliptic curves and Galois representations.

INT. JACOB SLEEPER HALL - BOSTON UNIVERSITY - DAY  
(AUGUST 9, 1995)

Approximately four hundred mathematicians are attending a conference on Wiles' proof. GLEN STEVENS gives the opening address.

## GLEN STEVENS

Welcome to the Instructional Conference on Number Theory and Arithmetic Geometry. The conference will focus on two major topics: (1) Andrew Wiles' recent proof of the Shimura-Taniyama Conjecture and (2) the earlier works of Frey, Serre, and Ribet showing that Wiles' theorem would complete the proof of the Fermat Conjecture. There will be more than twenty-five lectures during the conference.

Glen Stevens leaves the stage, and there are several days of lectures.

INT. LECTURE HALL ADJACENT TO JACOB SLEEPER HALL - BOSTON UNIVERSITY - DAY

CHRIS SKINNER and BRIAN CONRAD, both previous Ph.D. students of Andrew Wiles, are conducting a survey course on Andrew Wiles' proof of the Fermat Conjecture. Both are standing at the blackboard.

CHRIS SKINNER

Welcome to the course. During the next few days Brian Conrad and I will lecture in this room for two hours each day starting at 1:00 p.m. on Wiles' proof.

BRIAN CONRAD

Of course, we will only be able to give a sketch of the proof in the very limited time that we have.

CHRIS SKINNER

Our lectures will provide to anyone, who is willing to expend some effort, at least a glimpse of the basic ideas and components of the proof and of some of the people who were responsible for it.

INT. LOBBY OF JACOB SLEEPER HALL - BOSTON UNIVERSITY - DAY

SALESPERSON #1 is at a table selling a commemorative T-shirt. CUSTOMER #1 purchases one of the shirts.

CUSTOMER #1

What are you selling?

SALESPERSON #1

A commemorative T-shirt.

CUSTOMER #1

What's written on it?

SALESPERSON #1

On its front is displayed a sketch of the proof, with a single, ingeniously accurate, crucial paragraph. On its back appears a list of references to the fundamental papers by Frey, Serre, Ribet, Wiles and Wiles-Taylor.

CUSTOMER #1

OK...I'll buy one! Is it selling well?

SALESPERSON #1

Very well! In fact we may have to fill some orders by mail after the convention; when more shirts will be printed.

FEMALE MATHEMATICIAN #1 (about 45 years old) enters the lobby handing out her two-page proof of the Fermat Conjecture while proclaiming that Wiles' proof is not correct. Mathematician #1 confronts her.

MATHEMATICIAN #1

I overheard you say that Wiles' proof is not correct. Have you carefully read his paper?

FEMALE MATHEMATICIAN #1

No.

MATHEMATICIAN #1

Well then, how do you know it is not correct?

FEMALE MATHEMATICIAN #1

I just know it is not correct! Would you like a copy of my two-page proof of the Fermat Conjecture?

MATHEMATICIAN #1

Please take my advice and cease and desist what you are doing and throw the remaining copies of your proof into the wastebasket. You are just making a fool of yourself!

INT. DINING HALL ADJACENT TO JACOB SLEEPER HALL - BOSTON  
UNIVERSITY - DAY

Mathematician #6 and Barry Mazur are having lunch  
together.

MATHEMATICIAN #6

Barry, I have read Wiles' paper,  
and he employs many of your ideas  
and results. Did you try to  
establish the Shimura-Taniyama  
Conjecture after you heard of  
Ribet's result?

BARRY MAZUR

No...I was working on several  
projects when I learned that Ribet  
had reduced the proof of the  
Fermat Conjecture to a proof of  
the Shimura-Taniyama Conjecture.  
And then I had jury duty.

MATHEMATICIAN #6

Jury duty? What was that all  
about?

BARRY MAZUR

Well, in the fall of 1993 I was  
called to serve for six months on  
a jury for a murder trial. I had  
to arrange for others to teach my  
courses at Harvard while I was on  
that jury, and I had no time for  
research.

INT. JACOB SLEEPER HALL - BOSTON UNIVERSITY - DAY (AUGUST  
18, 1995)

ANDREW WILES

...and in concluding my lecture,  
which is the last lecture of the  
conference, I would like to thank  
Ken Ribet for providing me with  
seven years of work and for his  
level lowering-principle for  
modular forms, which I employed  
strategically in the proof. And  
if Gerhard Frey has any ideas  
about the Riemann Hypothesis, and  
if he would like to share them  
with us, we would be happy to  
listen.

Wiles is given a prolonged standing ovation.

FADE OUT

THE END